Subset of Formulas for Ray Tracing

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1 Introduction

These are key sets of formula for ray tracing from Jones and Stephenson (1975)

1.1 Reference

R. Michael Jones, Judith J. Stephenson,"A Versatile Three-Dimensional Ray Tracing Compute Program for Radio Waves in the Ionosphere," OT Report 75-75, US Department of Commerce, October 1975

2 Formulas

$$H(r, \theta, \varphi, k_r, k_\theta, k_\varphi) = \frac{1}{2} Re \left[\frac{c^2}{\omega^2}(k_r^2 + k_\theta^2 + k_\varphi^2) - n^2 \right] \tag{1}$$

$$\frac{dr}{d\tau} = \frac{\partial H}{\partial k_r} \tag{1}$$

$$\frac{d\theta}{d\tau} = \frac{1}{r}\frac{\partial H}{\partial k_\theta} \tag{1}$$

$$\frac{d\varphi}{d\tau} = \frac{1}{r\sin\theta}\frac{\partial H}{\partial k_\varphi} \tag{1}$$

$$\frac{dk_r}{d\tau} = -\frac{\partial H}{\partial r} + k_\theta \frac{d\theta}{d\tau} + k_\varphi \sin\theta \frac{d\varphi}{d\tau} \tag{1}$$

$$\frac{dk_\theta}{d\tau} = \frac{1}{r}(-\frac{\partial H}{\partial \theta} - k_\theta \frac{dr}{d\tau} + k_\varphi r \cos\theta \frac{d\varphi}{d\tau}) \tag{1}$$

$$\frac{dk_\varphi}{d\tau} = \frac{1}{r\sin\theta}(-\frac{\partial H}{\partial \varphi} - k_\varphi \sin\theta \frac{dr}{d\tau} - k_\varphi r \cos\theta \frac{d\theta}{d\tau}) \tag{1}$$

$$\frac{d\omega}{d\tau} = \frac{\partial H}{\partial t} \tag{1}$$

Table 1: List of Symbols

λ	Wavelength
λ_0	Wavelength Free Space
τ	Independent variable in Hamilton's Equations
φ	Longitude in spherical polar coordinates
ω	$2\pi f$, angular wave frequency
θ	Colatitude in spherical polar coordinates
k_r, k_θ, k_φ	Components of the propagation vector in the r,θ , φ directions – a vector perpendicular to the wave front having a magnitude $\frac{2\pi}{\lambda} = \frac{\omega}{v}$
f	Wave frequency
n	Phase refractive index (in general complex)
r,θ , φ	Coordinates of a point in spherical polar coordinates
s	Geometric ray path length
c	Speed of electromagnetic waves in free space.
t	Time, travel time of a wave packet.
ϵ_0	Electric permittivity of free space

$$R(1) = r \tag{1}$$

$$R(2) = \theta \tag{1}$$

$$R(3) = \varphi \tag{1}$$

$$R(4) = k_r \tag{1}$$

$$R(5) = k_\theta \tag{1}$$

$$R(6) = k_\varphi \tag{1}$$

Note R(7) through R(10) Variables the User Chooses to Integrate.

$$R(7) = P \text{ Phase Path in Kilometers} \tag{1}$$

$$R(8) = A \text{ Absorption in Decibels} \tag{1}$$

$$R(9) = \Delta f \text{ Doppler Shift in Hertz} \tag{1}$$

$$R(10) = s \text{ Geometrical Path Length in Kilometers} \tag{1}$$